Atelier franco-japonais de dynamiques réelles et complexes

Dates: From 9:00, September 11 to 18:00, September 15, 2023
Venue: Mon-Thu-Fri: Salle 2400 Tour Zamansky (24e étage)
Tue-Wed: Salle 502 couloire 15-25 (5e étage)
Sorbonne Université – Campus Pierre et Marie Curie
4, place Jussieu 75005, Paris

Program:

September 11 (Monday)

9:00 – 10:00	Romain Dujardin (Sorbonne)
10:30 – 11:30	Zin Arai (Tokyo Tech)
13:00 – 14:30	Open problem session
	Chairpersons: Hiroyuki Inou (Kyoto), Hiroki Takahasi (Keio)

September 12 (Tuesday)

9:00 – 10:00	Masato Tsujii (Kyushu)
10:30 – 11:30	Thomas Gauthier (Paris-Saclay)
13:00 – 14:00	Tomoki Kawahira (Hitotsubashi/Uppsala)
14:15 – 15:15	David Burguet (Sorbonne)
15:15 – 18:00	free discussion

September 13 (Wednesday)

9:00 – 10:00	Charles Favre (École Polytechnique)
10:30 – 11:30	Reimi Irokawa (NTT)
13:00 – 14:00	Marie-Claude Arnaud (Université de Paris Cité)
14:15 – 18:00	free discussion

September 14 (Thursday)

9:00 – 10:00	Jonguk Yang (Zürich)
10:30 – 11:30	Olga Paris-Romaskevich (Marseille)
13:00 – 14:00	Takato Uehara (Okayama)
14:15 – 15:15	Mats Bylund (Paris-Saclay)
14:15 – 18:00	free discussion
18:00 –	banquet

September 15 (Friday)

9:00 – 10:00	Thibault Lefeuvre (Sorbonne)
10:30 – 11:30	Mitsuhiro Shishikura (Kyoto)
13:00 – 18:00	free discussion

Titles and abstracts:

Romain Dujardin (Sorbonne)

Degenerate homoclinic bifurcations in complex dimension 2.

Unfolding homoclinic tangencies is the main source of bifurcations in 2-dimensional (real or complex dynamics). When studying this phenomenon, it is common to assume that tangencies are quadratic and unfold with positive speed. Adapting to the complex setting an argument of Takens, we show that any 1-parameter family of 2-dimensional diffeomorphisms unfolding an arbitrary tangency contains such quadratic tangencies. Combining this with the recent results of Avila-Lyubich-Zhang and former results in collaboration with Lyubich, this yields the abundance of robust homoclinic tangencies in the bifurcation locus for complex Hénon maps. We also study bifurcations induced by families with persistent tangencies, which give another approach to the complex Newhouse phenomenon.

Zin Arai (Tokyo Tech)

On the parameter space of the Hénon map.

We study the parameter space of the Hénon map using complex analytic techniques and computer-assisted proofs. We first introduce a rigorous algorithm to verify the uniform hyperbolicity of the map. This enables us to illustrate structurally stable parameter regions. Then, we focus on the largest stable parameter region, namely, the hyperbolic horseshoe region, and show that the first bifurcation that destroys the horseshoe is homoclinic or heteroclinic tangency. It follows that the hyperbolic horseshoe locus and the maximal entropy locus are connected and simply connected. By computing a topological invariant called the modulus of tangency, it is also shown that the dynamics of the Hénon map on the first bifurcation curve are mutually different; that is, there is no topological conjugacy between two parameter values on the curve.

Masato Tsujii (Kyushu)

Quasi-compactness of transfer operator and the viewpoint of micro-local analysis.

We discuss about the transfer operators for smooth dynamical systems from the viewpoint of micro-local analysis. We first explain the general idea of micro-local analysis and how it works in the study of the transfer operators for dynamical systems. This idea has been carried out successfully in the case of hyperbolic dynamical systems. In this talk, we would like to discuss how we can go further with the same idea into more general class of dynamical systems. As a simple instance, we will present a class of dynamical systems "virtually expanding dynamics" which are not necessarily (partially) hyperbolic and show that we can develop the argument for them exactly as in the case of hyperbolic dynamical

systems. Virtually expanding dynamical systems are open (and expected to be dense) in the space of volume-expanding dynamical systems on a closed manifold. Even though they are not necessarily (partially) hyperbolic, we show that the transfer operators of virtually expanding maps are quasi-compact on a (usual) Sobolev space. This implies that virtually expanding dynamical systems admits a finite system of physical measures. We expect that we observe interesting bifurcation phenomena of physical measures (or chaotic attractors) in the space of virtually expanding dynamical systems.

Thomas Gauthier (Paris-Saclay)

Entire and rational maps having integer multipliers

This talk is dedicated to the classification of entire and rational functions whose multipliers all lie in the ring of integers of an imaginary quadratic number field. Milnor conjectured in 2006 that such rational maps whose multipliers are Lattès maps, power maps or Chebichev maps. This conjecture was recently proved by Zhuchao Ji and Junyi Xie, relying on arguments from non-archimedean and arithmetic dynamics. In a joint work with Xavier Buff, Jasmin Raissy and Valentin Huguin, we gave a complex analytic proof allowing to generalize the result to transcendental entire functions. If time allows, I will present both the arithmetic and complex analytic proofs.

Tomoki Kawahira (Hitotsubashi/Uppsala)

Zalcman functions for holomorphic diffeomorphisms of $\mathbb{C}^2$.

We present a version of Zalcman's rescaling principle for the dynamics of holomorphic diffeomorphisms of $\mathbb{C}^2$. This will give a new insight to the notion of quasi-expansion by Bedford and Smillie.

David Burguet (Sorbonne)

Existence of maximal measures for C^r, $r < +\infty$, *smooth surface diffeomorphisms*

In the 70's, Misiurewicz built examples of C^r, $r < \infty$, diffeomorphisms without measures of maximal entropy. Building on Yomdin's theory, Newhouse proved much later that C^∞ systems always admit a measure of maximal entropy. We show that such maximal measures exist for C^r, $r < \infty$ surface diffeomorphisms with $h_{top}(f) > \frac{\log \|df\|_\infty}{r}$. The proof is based on an adaptation of the entropic continuity of Lyapunov exponents recently established by Buzzi, Crovisier and Sarig for C^∞ surface diffeomorphisms.

Charles Favre (École Polytechnique)

Entropy of Non-archimedean dynamical systems.

Reimi Irokawa (NTT)

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Marie-Claude Arnaud (Université de Paris Cité)

Conformal Symplectic Dynamics: some of their invariant sets from a point of view coming from dynamics, symplectic topology and PDE.

This is a joint work with Jacques Fejoz, Vincent Humilière and Claude Viterbo. Conformal Symplectic Dynamics alter the symplectic form up to a multiplicative constant that is not 1. We will study some of their invariant sets from different points of view: -dynamical systems: we will study their invariant submanifolds, focusing on their possible isotropy; -symplectic topology: under adequate hypotheses, we will state the existence of a unique invariant set that generalizes the Birkhoff attractor of twist maps; -PDE: under adequate hypotheses, we will prove a property of the graph of the differential of the viscosity solution of the discounted Hamilton-Jacobi equation.

Jonguk Yang (Zürich)

Renormalization and critical points for Hénon maps.

We generalize the renormalization theory of unimodal intervals maps to dimension two, so that it can be applied to the study of real Hénon maps. The key step is to identify the higher-dimensional analog of the critical orbit using a Pesin theoretic approach. As the main result, we will give an explicit description of the non-uniform partial hyperbolicity of Hénon maps that are properly dissipative, infinitely renormalizable and unimodal.

This is based on a joint work with S. Crovisier, M. Lyubich and E. Pujals.

Olga Paris-Romaskevich (Marseille)

Novikov's problem and tiling billiards

We are interested in the study of foliations defined by plane sections of surfaces inside the 3-torus and especially in a question asked by Novikov in the 80s : how big the set of minimal foliations is in the parameter space ? This topological question has a dynamical reformulation in terms of refractive billiards inside plane tilings, and corresponding families of interval exchange transformations.

We were able to advance in the answer to Novikov's question in the case of centrally symmetric surfaces of genus 3 : we give an explicit description of the set of minimal foliations given by a continued fraction algorithm coming from the renormalization process for tiling billiards in cyclic quadrilateral tilings.

The talk is based on a series of works, in collaboration with Ivan Dynnikov, Pascal Hubert, Paul Mercat and Alexandra Skripchenko.

Takato Uehara (Okayama)

On dynamical degrees of birational mappings.

For a birational mapping on a compact complex surface, one can define its dynamical degree, which measures the complexity of dynamical behavior of the mapping. In this talk, we will show some properties of the set of dynamical degrees.

Mats Bylund (Paris-Saclay)

Critical recurrence in the real quadratic family

The (real) quadratic family is one of the most well-studied families of dynamical systems, and has served as a representative model of chaotic dynamics for the last three decades. By works of Jakobson and Benedicks–Carleson, and later by Graczyk–Swiatek and Lyubich (to name only a fraction of names), this family of dynamical systems is now very well understood. In this talk I will discuss recurrence, and present a result which completes earlier estimates regarding the rate of recurrence for the critical point to itself for typical non-regular (stochastic, Collet–Eckmann) real quadratic maps.

Thibault Lefeuvre (Sorbonne)

Rapid mixing for extensions of Anosov flows

I will report on an ongoing project with Mihajlo Cekić (University of Zurich) aiming to understand speed of mixing for a class of partially hyperbolic flows obtained as extensions of Anosov flows to principal bundles.

I will explain one of the results obtained so far: a U(1)-extension of a volume-preserving Anosov flow (that is not a suspension) is super-polynomially mixing provided the U(1)-bundle is not torsion. In particular, this yields the following corollary: the frame flow over negatively-curved 3-manifolds is super-polynomially mixing.

If time permits, I will try to give an overview of possible future directions for this work: exponential mixing, generalization to other compact Lie groups, to Anosov actions, etc.

Mitsuhiro Shishikura (Kyoto)

Scaling limits of quadratic rational maps and trees of spheres.

When a family of rational maps degenerates, certain parametrized coordinate changes may give rise to a non-trivial return map. J. Kiwi studied such scaling limits for quadratic rational maps and M. Arfeux defined "trees of spheres" for the degeneration. We will discuss a converse problem which means a construction of degeneration family from a given data, and its relation to the Berkovich space of the extension of the field of Laurent series. This is a work in progress with Arfeux and Kiwi, and some work-out examples with E. Hironaka related to $\mathrm{Per}_n(0)$.

Organizers:

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